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WAITING TIME IN QUEUEING SYSTEM WITH DOUBLE REQUESTS

Consider a request divided into disconnected impulses, each requesting corresponding service on the same server (service line). For example, in computer networks any task is divided into frames, in air traffic control systems multiplicity of request is emerged due to intermittence of receiving information from airplanes and transmitting of control information to the airplane by flying control officer.

Queueing system

Consider a single-channel queueing system in which Poisson flow of requests arrives with rate λ. The service for each request consists of two servicing operations with constant duration τ, but the second operation can be started only after the first one is completed and after time $\tau+\Delta$ after the request entered a system. Division of operations' fulfillment is prohibited. As soon as request enters a system, dispatcher plans times for the first and second operations.

Let at moment of request's arrival, taken as zero, intervals $(0,x)$ and (y,z) are chosen to service the requests that have come earlier. If $y-x\geq\tau$ then the first operation of the new request takes interval $(x,x+\tau)$, otherwise $(z,z+\tau)$.

After time for the first request is planned, two intervals $(0,x^*)$ and (y,z^*) would be occupied, where can be two cases a) $x^*=x+\tau$, $y^*=y$, $z^*=z$, b) $x^*=x$, $y^*=y$, $z^*=z+\tau$. Then the second operation can be planned as: a) when $y^*-x^*\geq\tau$ and $x^*\geq\Delta+\tau$; b) when $x^*<\tau+\Delta$, $2\tau+\Delta\leq y^*$; c) when either $y^*-x^*<\tau$, or $2\tau+\Delta>y^*$, and $\tau+\Delta\leq z^*$; d) when either $y^*-x^*<\tau$, or $2\tau+\Delta>y^*$, and $z^*<\tau+\Delta$.

Problem statement:

1) find ergodicity conditions;
2) find distribution function $F(x)$ of virtual waiting time of double request;
3) find average waiting times w_1, w_2 of the first and second operations of random request, counting waiting time of the second operation from moment $\tau+\Delta$ after request entered the system.

Ergodicity conditions

Theorem 1. At $2\lambda\tau\geq 1$ the total waiting time $w_1^{(n)}+w_2^{(n)}$ of the first and second operations of the n-th request tends by probability to ∞ at $n\to\infty$.

Theorem 2. At $2\lambda\tau<1$ random vector $(w_1^{(n)},w_2^{(n)})$ has ergodic distribution.

Waiting time

Consider $\Delta<\tau$. In this case it will be impossible to place between operations of one request operation of another request; this simplifies analytical research. Let $w(t)$ be a virtual waiting time: the exact waiting time of request, if it arrives at time t. Main equations for distribution function $F(x)$ of virtual waiting time for double request

$$F'(z)-\lambda F(z)+\lambda F(z-2\tau)=0,\ z\geq 2\tau+\Delta,$$

$$F'(z)-\lambda F(z)=0,\ z<2\tau+\Delta.$$

Average waiting time

$$\mathrm{E}w_1=\mathrm{E}w=\frac{2\tau^2\lambda}{1-2\tau\lambda}+\Delta-\frac{1-2\tau\lambda}{\lambda}(1-e^{-\lambda\Delta}),$$

$$\mathrm{E}w_2=\frac{2\tau^2\lambda}{1-2\tau\lambda}.$$

Numeric results

Variant	Ew_1	Ew_2
λ=0.1; Δ=0	0.2500	0.2500
λ=0.1; Δ=0.5	0.3598	0.2500
λ=0.1; Δ=1	0.4887	0.2500
λ=0.2; Δ=0	0.6667	0.6667
λ=0.2; Δ=0.5	0.8812	0.6667
λ=0.2; Δ=1	1.1229	0.6667