State-Dependent Opinion Dynamics

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Studies of opinion formation in a network setting have traditionally assumed a fixed network topology over which the spread of opinions is studied. However, because agents form and sever links with each other based on their agreement or disagreement, and because network structure accentuates some interactions over others and therefore some opinions over others, network and opinion formation should not be studied as two independent processes. A more realistic setting requires modeling the interaction between these two processes in order to understand how they co-evolve. This understanding in turn brings about more opportunities for controlling the combined system. The main departure in our work from previous models is the focus on the interplay between these two processes and their simultaneous evolution.

Our starting point is a parametrized model that captures a wide range of interaction behavior. We consider a society of N individuals, each having an opinion normalized to be in $[0, 1]$. The society has the following dynamics: at each time step, each agent selects another agent in the population and forms a link with them (we will refer to this as a visit). The visiting agent then updates his opinion by taking a weighted average of his opinion and the opinion of the agent that he visited. Agent i's decision to visit agent j depends on two things, a) how similar j's opinion is to i, and b) how popular agent j is, where popularity is defined by the number of visits that j has received in the past. The process repeats indefinitely.

Our model captures two important aspects of a society through two non-negative parameters, β and κ. β is a parameter that measures how open a society is to opinions that are different from the norm within that society, with higher values of β indicating more conservative societies. κ is a parameter that describes the tendency of a society to follow the opinions of popular individuals. Formally, if we denote agent i's opinion at time t by $x_i(t)$ and agent j's in-degree by $d_j(t)$, then the probability with which agent i visits agent j at time t is given by

$$p_{ij}(t) = \frac{e^{-\beta|x_i(t)-x_j(t)|+\kappa d_j(t-1))}}{\sum_{k\neq i} e^{-\beta|x_i(t)-x_k(t)|+\kappa d_k(t-1))}}$$

where the term $d_j(t-1)$ indicates the number of visits agent j received in the previous time step, which we use as a proxy of how popular agent j is.

Unlike previous work, the evolution of the network topology in our model cannot be described by a stochastic process with independent and identically distributed increments and does not possess any exchangeability properties. Despite this, we are able to show that the system converges to a consensus in opinion. The rate of convergence, as well as the limiting opinion, heavily depend on

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the values of the parameters β and κ. We show how these parameters interact and how they affect the evolution of, and opinions in, the network.