STOCHASTIC ORDERING OF NETWORK THROUGHPUTS USING FLOW COUPLINGS

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Extended abstract

Robust estimates for the performance of a complex queueing network can be obtained by showing that the number of jobs in the network is stochastically comparable to a simpler, analytically tractable reference network [8]. Classical coupling results on the stochastic ordering of the network populations require strong monotonicity assumptions [3, 6, 7, 9] which are often violated in practice. However, in most real-world applications we care more about what goes through a network than what sits inside it. This poster describes a new approach [5] for ordering *flows instead of populations* by augmenting the network states with their associated flow-counting processes and developing order-preserving couplings [1, 6] of the augmented state–flow processes. This technique, closely related to the theory of monotone generalized semi-Markov processes developed by Glasserman and Yao [4], may also be regarded as a probabilistic analogue of the Markov reward comparison method in [2].

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