Belief propagation for optimal edge cover in the random complete graph

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Abstract

We apply the objective method of Aldous [1, 2] to the problem of finding the minimum cost edge-cover of the complete graph with random independent and identically distributed edge-costs. The limit, as the number of vertices goes to infinity, of the expected minimum cost for this problem is known via a combinatorial approach of Hessler and Wästlund [5] with a different proof by Wästlund [7]. We prove the same result using the machinery of the objective method and local weak convergence developed by Aldous in [1, 2]. This method was used to prove the $\zeta(2)$ limit of the random assignment problem. We further show that a belief propagation algorithm, similar to the work of Salez and Shah [6], converges to the optimal solution asymptotically.

The objective method appears powerful enough to be applied to several combinatorial probability problems. See [4] for a survey. However, the proof of optimality in [2] appears to be linked closely with the combinatorial specifics of the minimum matching problem. Our proof in the edge-cover problem exploits the form of *the recursive distributional equation* [3] rather than the combinatorics of the problem. Our approach may serve to make the objective method more generally applicable to other problems.

Along the lines of [6] we establish that the update rule for belief propagation on the finite complete graph converges in law to a limiting update rule on the limiting infinite tree. The latter update rule is shown to converge to a unique configuration that solves a recursive distributional equation associated with the edge-cover problem on the infinite tree. We then prove that this solution is optimal for the edge-cover problem on the infinite tree. Local weak convergence then establishes the asymptotic optimality of the algorithm.

The complexity of the belief propagation algorithm is $O(n^2)$ when compared with the worst case complexity of $O(n^3)$.

References

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