Extended abstract: A dynamic model of kidney exchange programs

Ross Anderson* Itai Ashlagi† David Gamarnik‡

May 1, 2012

Background: End stage renal disease, failure of the kidneys, is a serious problem in the United States and throughout the world. There are two primary treatments: dialysis, where patients are connected to a machine that filters the blood as a healthy kidney would for 12 or more hours per week, and alternatively a kidney transplant. The latter option results in lower mortality rates, higher patient quality of life, and less expense. However, the demand for kidneys to transplant greatly exceeds the supply. There are two sources of transplanted kidneys, kidneys from deceased donors and kidneys from living donors (healthy individuals can live safely with only one of their two kidneys). We focus on the allocation of kidneys given by living donors.

Living donor kidneys are allocated through specialized exchange programs. In these programs, there are two categories of donors: *directed donors* that want to give their kidney to a friend or family member, and *altruistic donors* that are willing to give away their kidney and receive nothing in return. When a directed donor is compatible with their patient, usually they simply transplant the pair and they never enter the exchange program. Otherwise, the incompatible pair enters the exchange program to be matched in a *cycle* of two, three or even four incompatible pairs, so that each pair gives and receives one kidney (generally cycles of length greater than four are avoided as the logistics of performing so many simultaneous operations are complex). Alternatively, the altruistic donors are used in these exchange programs to start *chains*. In a chain, an altruistic donor gives their kidney to an incompatible pair, then the donor from that pair gives their kidney to another incompatible pair, continuing until the chain is either broken (for example, by a donor that reneges) or ended with a patient that has no donor.

There are several living donor kidney exchange programs in the US. Each exchange uses their own algorithm to determine what cycles to match and how to direct chains, with the general goal of finding everyone a high quality transplant as quickly as possible. Currently, the existing exchanges are either waiting a fixed time period and then computing the maximum number exchanges they can make, or they are just greedily matching pairs after each arrival if it is possible to create a cycle or extend a chain. These exchanges are not regulated by the government, and some are run for profit. As patients are listed on multiple exchanges, there are incentives for the exchanges to match patients as quickly as possible as well. This is unlike deceased donor transplants, which the government controls in a centralized system.

A central question is, how do we optimize this allocation process in way that is practically implementable given the existing system and the incentive structures? It is well understood how to solve the "one shot" optimization problem of computing a maximum cardinality cycle packing [1] or a maximum cardinality cycle and chain packing [2] using integer programming. However, in reality donor patient pairs arrive dynamically, and want to be matched as quickly as possible. We take an important step towards answering this central question by addressing the narrower question: Within the class of policies where the "one shot" problem is solved periodically, which policy minimizes the average time donor patient pairs must wait to be matched?

The problem: We are interested in determining what policy the exchanges should use in forming chains and cycles to minimize the average time pairs must wait to be matched. We consider the class of policies where every t days, a maximum cardinality matching of cycles and chains is computed. Note that as $t \to 0$, we obtain the greedy policy.

A dynamic model: We create a stylized model to show that the greedy policy performs poorly within the class of "one shot" policies. The model considers only cycles, not chains, and assumes only tissue type incompatibilities, not blood type incompatibilities. The model has three parameters: p, the probability that a donor and recipient are compatible, t, the number of time periods between matchings, and k, the maximum size cycle. In each time period, a new incompatible donor-recipient pair arrives, and is represented by a node in a directed graph. A directed edge representing compatibility is drawn from each existing incompatible pair to the new pair with probability p, and the reverse edge is drawn with probability p as well, where each edge occurs independently. Then every t periods, a maximum cardinality cycle packing using cycles with at most k nodes is computed, and these nodes are removed from the graph. We prove that for every k, under the greedy policy ($t = 1$), the long run average waiting time is $\Theta(1/p^2)$. Alterna-

*Operations Research Center, MIT, rma350@mit.edu

†Assistant Professor of Operations Research, Sloan School of Management, MIT, iashlagi@mit.edu

‡Associate Professor of Operations Research, Sloan School of Management, MIT, gamarnik@mit.edu

tively, for the policy where $t = 1/p^{1+1/k}$, using a result from [2], we prove that the long run average waiting time is $\Theta(1/p^{1+1/k}) < \Theta(1/p^2)$, thus demonstrating that the greedy policy is suboptimal by an order of magnitude.

Empirical evidence: We use historical data from an existing kidney exchange to compare all of the "one shot" policies. In particular, we explore the impact of greedy policies on matching highly sensitized patients (patients with tissue type that is incompatible with the vast majority of donors), which are not captured in or stylized model.

References

[1] D.J. Abraham, A. Blum, and T. Sandholm. Clearing algorithms for barter exchange markets: enabling nationwide kidney exchanges. In *Proceedings of the 8th ACM conference on Electronic commerce*, pages 295–304. ACM, 2007.

[2] I. Ashlagi, D. Gamarnik, and A. Roth. The need for chains in kidney exchange. NBER Market Design Workshop, 2011.