Auctions with a Profit Sharing Contract

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[Extended Abstract]‡

We study the problem of selling a resource through an auction mechanism. The winning buyer in turn develops this resource to generate profit. Once a buyer develops the resource, the profit may become known to both the buyer and the seller through observable elements such as sales data, quality of the resource, market condition, etc. This raises the possibility of the seller selecting ex-ante as part of his auction a contract that specifies how the net profit will be split ex-post between the winning buyer and the seller.

We compare the following two forms of payment: (i) the seller either charges the winning buyer a one-time payment at the end of the auction stage; or (ii) the seller receives an initial payment from the winning buyer at the end of the auction stage followed by a *profit-sharing contract* (henceforth, PSC) in which he additionally receives a prespecified share of the realized profit from the resource. This is motivated by a current example: the FCC spectrum auctions (e.g., auction 73) are of the first type, while the 3G spectrum auctions in India require that a winning buyer pay a spectrum usage charge equal to a percentage of his profit in addition to the spectrum acquisition fees. We investigate whether or not there are economic reasons for the seller to prefer auctions with a PSC over auctions with only a one-time payment. The solution to this problem is nontrivial because strategic buyers adjust their bids in the auction stage in response to the payment they are required to make according to the PSC.

Our prime focus is on two simple PSCs. First is a *profit only sharing contract* (henceforth, POSC) where the seller takes a fixed fraction $\alpha \in (0,1)$ of the positive profit from the winning buyer but does not take any negative profit (loss) from him. Second is a *profit and loss sharing contract* (henceforth, PLSC) where the seller takes a fraction $\alpha \in (0,1)$ of both positive and negative profit from the winning buyer. We consider a symmetric interdependent values model of [1] with risk averse or risk neutral buyers and a risk neutral seller. For the second price and the English auctions, we show that:

(i) The seller's expected total revenue from the auction with a POSC or a PLSC is nondecreasing in the share fraction α, and in particular, is higher than the expected revenue from the auction with only a one-time payment.

(ii) For the same share fraction α, the auction with a PLSC generates higher expected total revenue than the auction with a POSC.

(iii) Moving beyond simple PSCs, we show that the auction with a PSC from a very general class generates higher expected total revenue than the auction with only a one-time payment. Moreover, the PLSC with the share fraction α is revenue optimal over the general class of PSCs for which the seller's marginal share is bounded by α.

(iv) Finally, we consider the issue of providing incentives to the winning buyer for developing the resource using a principal-agent relationship; we show that the auction with a PLSC and a suitably small share fraction α generates higher expected total revenue than the auction with only a one-time payment.

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‡The full version of this paper is available at http://arxiv.org/abs/1102.3195

References

[1] P. R. Milgrom and R. J. Weber, "A theory of auctions and competitive bidding," *Econometrica*, vol. 50, no. 5, pp. 1089–1122, 1982.